Australian Journal of Basic and Applied Sciences, 9(13) Special 2015, Pages: 19-25

ISSN:1991-8178

Australian Journal of Basic and Applied Sciences

Journal home page: www.ajbasweb.com

Effect of Environmental Factors on the Transmission of Chikungunya Fever

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ARTICLE INFO

Article history:
Received 22 February 2015
Accepted 20 March 2015
Available online 23 April 2015

Keywords:
Mathematical model, Chikungunya fever, Stability analysis, Basic reproductive number, Environmental factors

ABSTRACT

A mathematical model for the transmission dynamics of Chikungunya fever, caused by the infected mosquito feeding on the susceptible human is proposed and analyzed. The model takes into account the effect of environmental factors in the model. The modeling process, assumed that the growth rate of mosquito population is increate due to environment discharges caused by human population relates factors. The disease free equilibrium and endemic equilibrium state are determined and stabilities are analyzed. Basic reproductive number is found by the next generation matrix and the spectral radius. The results show that if the growth rate of the mosquito population caused by the environmental factors increase, the spread of Chikungunya fever increases. Numerical simulations are also carried out to investigate the influence of certain parameters on the spread of disease, to support the analytic results of the model.

To Cite This Article: Sumalee Srichan and Surapol Naowarat, Effect of Environmental Factors on the Transmission of Chikungunya Fever. ***Aust. J. Basic & Appl. Sci.***, *9(13): 19-25, 2015*

INTRODUCTION

Chikungunya fever is an arboviral disease transmitted to human by biting of *Aedes aegypti* mosquitoes and *Aedes albopictus* mosquitoes. The Chikungunya virus (CHIKV) is a member of the genus *alphavirus* and the *Togavivirus* family (Pioloux *et al* 2007). The symptoms of this disease are a high fever, extreme joint paints, and an irritating skin rash. Symptoms are very similar to dengue fever but there is no leakage of plasma out of blood vessel (Massad *et al* 2008). This disease is self- limiting and rarely fatal (Hochedez *et al*, 2006). There is no vaccine, infected human can be treated by drugs related symptom. In Asia, Chikungunya fever is occurred in many countries; Philippines (1954,1956 and 1968), India(1964), Sri Lanka(1969), Vietnam(1975), Indonesia (1973 and 1982) and Thailand(1958 and 2008) (Medlock, *et al* 2006). There are 2 billion people living in *Aedes*- infested areas.(WHO,2006)

Mathematical models have become an important tool for understanding the spread and control of disease. Misra and Mishra (2012) proposed mathematical model for Chikungunya fever with the effects of control measures have been taken into account in the model. It found that the disease free equilibrium point is locally asymptotically stable when control reproductive number $R_C < 1$ and unstable when $R_C > 1$. It have also found that the endemic equilibrium point is locally asymptotically stable when $R_C > 1$. Domont, Chiroleu and Domerg (2008) formulated a mathematical model for human and mosquito population for Chikungunya fever by SEIR model and found the stability conditions for disease free and endemic states. Naowarat *et al* (2011) proposed a model of Chikungunya transmission in which adulticide was used as an intervention tool to control the spread and found the conditions for disease free and endemic states.

Model formulation:

In our model, we assume that the human population is constant denote by N_h. The human population is divided into three compartments; the susceptible human $(\bar{S}_h)$, the infected human $(\bar{I}_h)$ and the recovered human $(\bar{R}_h)$; the mosquito population is divides into two compartments ,the susceptible mosquito $(\bar{S}_m)$,the infected mosquito $(\bar{I}_m)$ and the environmental factors denoted by $\bar{E}$. As shown in Fig. 1

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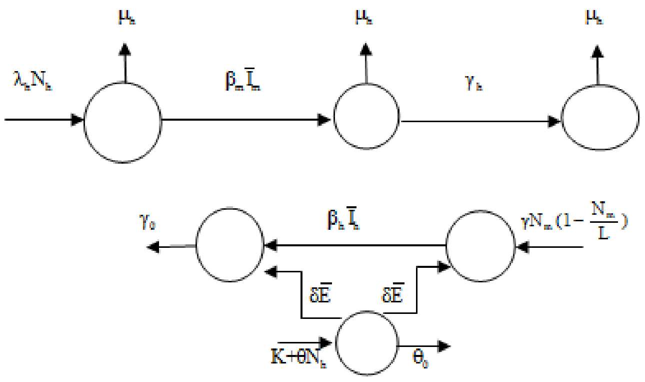


Fig. 1: Flow chart of the dynamical transmission of Chikungunya fever.

We defined,

$\bar{S}_h$ is the number of susceptible human population at time t

$\bar{I}_h$ is the number of infected human population at time t

$\bar{R}_h$ is the number of recovered human population at time t

$\bar{S}_m$ is the number of susceptible mosquito population at time t

$\bar{I}_m$ is the number of infected mosquito population at time t

$\bar{E}$ is the cumulative density of environmental discharges conducive to the growth of mosquito population at time t;

The transmission model will be a system of ordinary differential equations given by,

$$\frac{d\bar{S}_h}{dt} = \lambda_h N_h - \beta_m \bar{I}_m \bar{S}_h - \mu_h \bar{S}_h \qquad (1)$$

$$\frac{d\bar{I}_h}{dt} = \beta_m \bar{I}_m \bar{S}_h - (\mu_h + \gamma_h)\bar{I}_h \qquad (2)$$

$$\frac{d\bar{R}_h}{dt} = \gamma_h \bar{I}_h - \mu_h \bar{R}_h \qquad (3)$$

$$\frac{d\bar{S}_m}{dt} = \gamma N_m\left(1-\frac{N_m}{L}\right) - \beta_h \bar{S}_m \bar{I}_h - \gamma_0 \bar{S}_m + \delta\bar{E}\bar{S}_m \qquad (4)$$

$$\frac{d\bar{I}_m}{dt} = \beta_h \bar{S}_m \bar{I}_h - \gamma_0 \bar{I}_m + \delta\bar{E}\bar{I}_m \qquad (5)$$

$$\frac{d\bar{E}}{dt} = K + \theta N_h - \theta_0 \bar{E} \qquad (6)$$

where

λ_h is the birth rate of human population,

μ_h is the natural death rate of human population and death induced the disease,

β_m is the probability that virus transmitted from

3.

infected mosquito to susceptible
human population,

β_h is the probability that virus transmitted from infected human to susceptible
mosquito population

γ_h is the recovery rate of infected human population

γ is the growth rate of mosquitoes population

γ_0 is the natural death rate and control measure of mosquito population

L is the carrying capacity of mosquito population in the natural
environment;

δ is the per capita growth rate coefficients of mosquito population due to
conductive environmental discharge,

K is the cumulative density of environmental discharge grows due to constant influx rate,

θ is the cumulative density of environmental discharge grows due to human activities,

θ_0 is the depletion rate coefficient of the environmental discharges.

Model analysis:

Equilibrium Points:

1. Disease free equilibrium point (E_0): there are no infected human and infected mosquito , that is . $I_h = 0, I_m = 0$. Substituting this in equations (1)-(6), we obtained

$$E_0(S_h, I_h, I_m, E) = \left(1,\ 0,\ 0,\ \frac{K+\theta N_h}{\theta_0}\right).$$

2. Endemic equilibrium point (E_1): In case $I_h \neq 0, I_m \neq 0$, we obtained

$$E_1\left(S_h{}^*, I_h{}^*, I_m{}^*, E^*\right) = \left(\frac{\lambda_h}{\beta_m N_m I_m{}^* + \mu_h},\ \frac{\lambda_h N_m \beta_m I_m{}^*}{(\mu_h+\gamma_h)\left(\beta_m N_m I_m{}^* + \mu_h\right)},\ I_m{}^*,\ \frac{K+\theta N_h}{\theta_0}\right) \text{ where}$$

$$\mathrm{I}_\mathrm{m}^* = \frac{\lambda_h N_h \beta_h \beta_m N_m - \mu_h(\mu_h+\gamma_h)\left(\gamma_0 - \delta(\frac{k+\theta N_h}{\theta_0})\right)}{\lambda_h N_h \beta_h \beta_m N_m + \beta_m N_m(\mu_h+\gamma_h)\left(\gamma_0 - \delta(\frac{k+\theta N_h}{\theta_0})\right)}.$$

Basic Reproductive number:

We obtained a basic reproductive number $(\Re_0)$ by using the next generation matrix (van den Driessche and Watmough,2002). Rearrange the equation (7),(8),(9),(10) in matrix form

$$\frac{dx}{dt} = F(x) - V(x)$$

$$\mathrm{F}(\mathrm{x}) = \begin{bmatrix} 0 \\ \beta_m N_m I_m S_h \\ \beta_h N_h (1 - I_m) I_h \\ 0 \end{bmatrix}, \quad \mathrm{V}(\mathrm{x}) = \begin{bmatrix} \mu_h S_h - \lambda_h \\ (\mu_h + \gamma_h) I_h \\ \gamma_0 I_m - \delta E I_m \\ \theta_0 E - K - \theta N_h \end{bmatrix}$$

Find the Jacobian of $F(x)$ and $V(x)$, denoted by $DF(x) = F$ and $DV(x) = V$, we obtained

$$\mathrm{F} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \beta_m N_m I_m & 0 & \beta_m N_m S_h & 0 \\ 0 & \beta_h N_h (1 - I_m) & -\beta_h N_h I_h & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \mathrm{V} = \begin{bmatrix} \mu_h & 0 & 0 & 0 \\ 0 & \mu_h + \gamma_h & 0 & 0 \\ 0 & 0 & \gamma_0 - \delta E & -\delta I_m \\ 0 & 0 & 0 & \theta_0 \end{bmatrix}$$

Find F and V at $\mathrm{E}_0 = \left(1, 0, 0, \frac{K + \theta N_h}{\theta_0}\right)$

We obtained

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_m N_m & 0 \\ 0 & \beta_h N_h & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} \mu_h & 0 & 0 & 0 \\ 0 & \mu_h + \gamma_h & 0 & 0 \\ 0 & 0 & \gamma_0 - \delta E & 0 \\ 0 & 0 & 0 & \theta_0 \end{bmatrix}$$

Find FV^{-1}

$$FV^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{\beta_m N_h}{\gamma_0 - \delta E} & 0 \\ 0 & \dfrac{\beta_h N_m}{\mu_h + \gamma_h} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Find spectral radius of FV^{-1} denoted by $\rho\left(FV^{-1}\right)$

$$\rho\left(FV^{-1}\right)=\begin{vmatrix} -\lambda & 0 & 0 & 0 \\ 0 & -\lambda & \dfrac{\beta_m N_m}{\gamma_0-\delta E} & 0 \\ 0 & \dfrac{\beta_h N_h}{\mu_h+\gamma_h} & -\lambda & 0 \\ 0 & 0 & 0 & -\lambda \end{vmatrix}$$

We obtained

$$\Re_0 = \sqrt{\frac{\beta_m\beta_h N_m N_h}{(\mu_h+\gamma_h)(\gamma_0-\delta E)}}$$

Local stability:

The local stability of an equilibrium point is determined from the Jacobian matrix of the system of ordinary differential equation ()-() evaluated at the equilibrium point. The Jacobian matrix at E_0 is

$$J_0=\begin{bmatrix} -\mu_h & 0 & -\beta_m N_m & 0 \\ 0 & -(\mu_h+\gamma_h) & \beta_m N_m & 0 \\ 0 & \beta_h N_h & -(\gamma_0-\delta E) & 0 \\ 0 & 0 & 0 & -\theta_0 \end{bmatrix}$$

The eigenvalues of the J_0 are obtained by solving $|J-\lambda I|=0$. We obtained the characteristic equation,

$$(-\theta_0-\lambda)(-\mu_h-\lambda)\left(\lambda^2+A\lambda+B\right) \qquad =0$$

where

$$A=\mu_h+\gamma_h+\gamma_0-\delta E$$
$$B=(\mu_h+\gamma_h)(\gamma_0-\delta E)-\beta_h\beta_m N_m N_h$$

From the characteristic equation, We see that two are $\lambda=-\theta_0<0, \lambda=-\mu_h<0$. The other two eigenvalues are solution of $\lambda^2+A\lambda+B$. The roots of this equation will be negative if two coefficients satisfied with the Routh-Hurwitz criteria.

1) $A>0$
2) $B>0$.

Endemic equilibrium point:

To determine the stability of the endemic equilibrium point, E_1 by examining the eigenvalues of Jacobian matrix at E_1, which is

$$J_1=\begin{bmatrix} -(\beta_m N_m I_m+\mu_h) & 0 & -\beta_m N_m S_h & 0 \\ \beta_m N_m I_m & -(\mu_h+\gamma_h) & \beta_m N_m S_h & 0 \\ 0 & \beta_h N_h (N_m-I_m) & -(\beta_h N_h I_h+\gamma_0-\delta E) & \delta I_m \\ 0 & 0 & 0 & -\theta_0 \end{bmatrix}$$

$\lambda_1=-\theta_0$ and $\lambda^3+C_1\lambda^2+C_2\lambda+C_3 \ = \ 0$

Where

$$
\begin{aligned}
C_1 &= \beta_m N_m I_m^* + \mu_h + \mu_h + \gamma_h + \beta_h N_h I_h^* + \gamma_0 - \delta E \\
C_2 &= \left(\beta_m N_m I_m^* + \mu_h\right)\left(\mu_h + \gamma_h + \beta_h N_h I_h^* + \gamma_0 - \delta E\right) \\
&\quad + \left(\mu_h + \gamma_h\right)\left(\beta_h N_h I_h^* + \gamma_0 - \delta E\right) - \beta_h N_h \beta_m N_m S_h^* \left(1 - I_m^*\right) \\
C_3 &= \left(\beta_m N_m I_m^* + \mu_h\right)\left(\mu_h + \gamma_h\right)\left(\beta_h N_h I_h^* + \gamma_0 - \delta E\right) - \beta_h N_h \beta_m N_m S_h^* \left(\beta_m N_m I_m^* + \mu_h\right)\left(1 - I_m^*\right) \\
&\quad + \beta_h N_h S_h^* I_m^* \left(\beta_m N_m\right)^2 \left(1 - I_m^*\right)
\end{aligned}
$$

The three eigenvalues of $\lambda^3 + C_1\lambda^2 + C_2\lambda + C_3 = 0$ will be negative real part if they satisfy the Routh-Hurwitz criteria.

1) $C_1 > 0,$
2) $C_3 > 0,$
3) $C_1 C_2 > C_3.$

Numerical results:

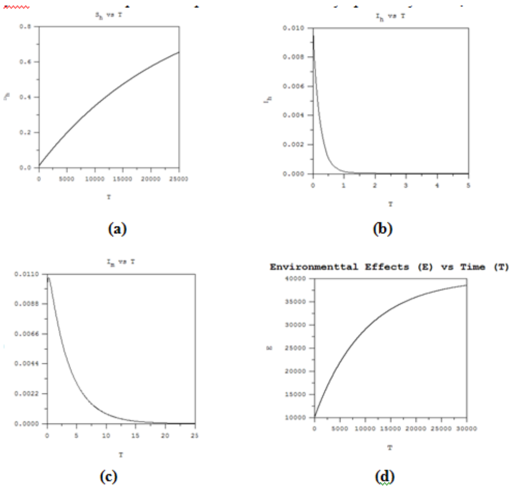
We solved the system of ordinary differential equations. The numerical results showed that the numerical results obtained from simulation are presented in the following fig.2 and 3.

Stability of the disease free state:

From the values of parameters in Table 1, we obtained the eigenvalues and basic reproductive number are:

$\lambda_1 = -0.0001, \lambda_2 = -0.000042, \lambda_3 = -0.012576, \lambda_4 = -8.74747, R_0 = 0.950222, \Re_0 = 0.974793 < 1.$

Since all of eigenvalues are to be negative and the basic reproductive is to be less than one,the disease free equilibrium point will be local asymptotically stable, as shown in Fig. 2



(a) (b)

(c) (d)

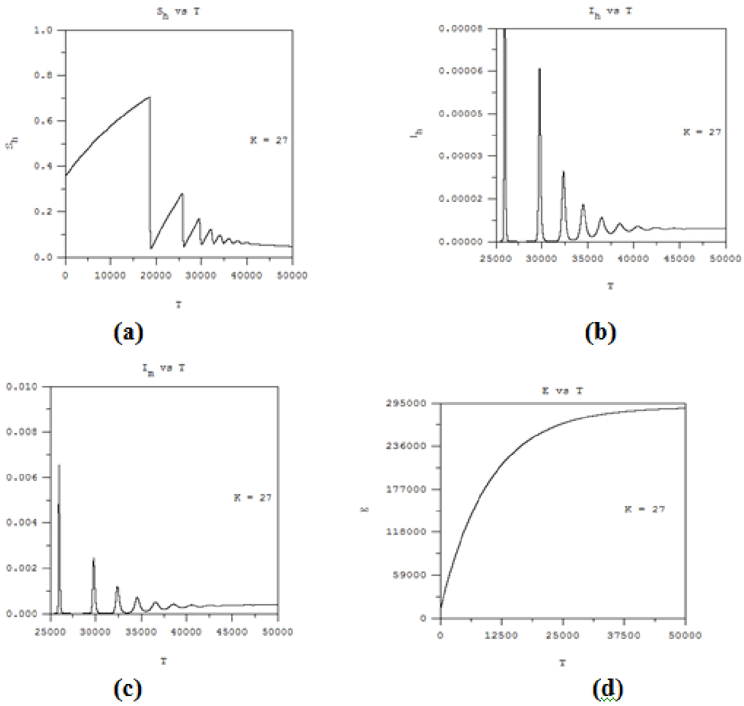
Fig. 2 Time series of (a) Susceptible human (S_h), (b) Infected human (I_h), (c) Infected mosquito (I_m), (d) Environment (E). The values of parameters are in the text and $R_0 < 1$. We see that the solutions converge to the disease free state $E_0(1, 0, 0, 40000)$, where $N_h = 10000$, $N_m = 7000$, $\mu_h = 0.000042$, $\beta_m = 0.0003$, $\beta_h = 0.0001$, $\gamma_h = 8.5$, $\gamma = 0.6$, $\gamma_0 = 0.3$, $L = 1000$, $\delta = 0.000001$, $K = 2$, $\theta = 0.0002$, $\theta_0 = 0.0001.$

Stability of the endemic state:

We changed the values of k=27 and keep the values of the other values of parameters to be those given in Table 1, we obtained the eigenvalues and basic reproductive number are:

$\lambda_1 = -0.0001, \lambda_2 = -0.010058, \lambda_3 = -8.58004, \lambda_4 = -0.000989, R_0 = 24.7058, \Re_0 = 4.97049 > 1.$

Since all of eigenvalues are to be negative and the basic reproductive is to be greater than one, the endemic equilibrium point will be local asymptotically stable, as shown in Fig. 3



(a) (b)

(c) (d)

Fig. 3: Time series of (a) Susceptible human (S_h), (b) Infected human (I_h), (c) Infected mosquito (I_m), (d) Environment (E). The values of parameters are in the text and $R_0 > 1$. We see that the solutions converge to the disease free state $E_1(0.040496, 0.000005, 0.000474, 290000)$, where $N_h = 10000$, $N_m = 7000$, $\mu_h = 0.000042$, $\beta_m = 0.0003$, $\beta_h = 0.0001$, $\gamma_h = 8.5$, $\gamma = 0.6$, $\gamma_0 = 0.3$, $L = 1000$, $\delta = 0.000001$, $K = 27$, $\theta = 0.0002$, $\theta_0 = 0.0001$.

Discussion:

In our finding, there exist a disease free equilibrium state. We shows that if $R_0 < 1$, then the disease free equilibrium point is local asymptotically stable, that is the disease will died out from the community. We also showed that an endemic equilibrium state exist when $R_0 > 1$, then the endemic equilibrium point is local asymptotically stable, that is the disease will occur in the community. We obtained the basic reproductive number through the use of spectral radius of the next generation matrix . The basic reproductive is $\Re_0 = \sqrt{R_0}$, where $R_0 = \dfrac{\beta_m \beta_h N_m N_h}{(\mu_h + \gamma_h)(\gamma_0 - \delta E)}$. Our numerical simulations demonstrated that $\Re_0$ will be increase when the k increase. We found the value of $\Re_0 = 0.974793,\ 2.02919,\ 4.97049$ when $k = 2,\ 22,\ 27$ respectively. It seen that the infected human will increase when the increase of factor of environmental with support for mosquitoes populations (Naresh and Pandey, 2009).

ACKOWLEGEMENT

The authors are grateful to the Department of Mathematics, Faculty of Science and Technology, Suratthani Rajabhat University, Surat Thani, Thailand for providing the facilities to do the research.

REFERENCES

Dumont, Y., F. Chiroleu, C. Domerg, 2008. On the temporal model for the Chikungunya disease: Modeling, theory and numerics, *Math Biosci*, 213: 80-91.

Environmental Effects. *AAM*., 4: 155-175.

Hochedez, P., S. Jaureguiberry, Debruyne M, 2006. Chikungunya infection in travelers, *Emerg Infect Dis*, 12: 1565-1567.

Massad, E., S. Ma, MN. Burattini, Y. Tun, FB. Coutinho, LW. ANG, 2008. The Risk of Chikungunya Fever in a Dengue-Endemic Area. *J Travel Med*, 15(3): 147-155.

Medlock, JM., D. Avenell, I. Barrass, S. Leach, 2006. Analysis of the potential for survival and seasonal activity of *Aedes albipictus* (Diptrera:Culicidae) in the United Kingdom, *Journal* *of Vector Ecology*, 31(2): 292-304.

Naowarat, S., W. Tawarat, IM. Tang, 2010. Control of the transmission of chikungunya fever epidemic through the use of adulticide, *Am J Applied Sci*, 8(6): 558-565.

Naresh, R., S. Pandey, 2009. Modeling and Analysis of the Spread of Japanese Encephalitis with

Pioloux, G., BA. Gauzere, S. Janureguiberry, M. Strobel, Chikungunya, 2007. An epidemic arbovirosis *Lancet Infect Dis.*, 7: 319-327.

Van den Driessche, P., J. Watmough, 2002. Reproductive numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Math Biosci*, 180: 29-48.

World Health Organization, 2006. Chikungunya and dengue in the south west Indian Ocean. Available at:
http://www.seaaaaaaro.who.int/LinkFiles/Dengue_chap-7.pdf.(Acessed 2006 Mar17)